A New Method for Measuring Dielectric Constant Using the Resonant Frequency of a Patch Antenna

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Abstract —An analytical expression is given for the resonant frequency of a rectangular patch antenna. It shows explicitly the dependence of the resonant frequency on the characteristic parameters of a patch antenna. Based on this result, a new method is developed for the measurement of the dielectric constant of a thin slab substrate. Basically, the test equipment consists of a rectangular microstrip antenna, the patch of which is fed either by a microstrip line or coaxial line. From the measured resonance parameters of the rectangular patch antenna, the dielectric constant can be easily obtained. The measured values of the present method are in agreement with the precision standard cavity resonator method. Accuracy of the dielectric constant so obtained is satisfactory.

I. Introduction

FOR 40 years, the closed-cavity resonator operated in the low-attenuation TE_{01} mode has been used to measure the dielectric constant of materials in the microwave range [1], in particular that of low-loss solids. Reviews of the literature on dielectric measuring methods including cavity resonator techniques have been given by Lynch [2], and, more recently, by Birch and Clarke [3].

In designing microwave-integrated circuits, accurate values of the dielectric constant of the substrate material must be known. A typical method for measuring the dielectric properties of the substrate material is the one reported by Itoh [4]. In the case of a microstrip cavity, both sides of the substrate plate are first metallized to form a parallel-plate resonator, and the measurement of the resonant characteristic of such a resonator leads to the desired quantity.

Another method mentioned in [5] has been investigated, namely the microstrip antenna method as a nondestructive method for the measurement of the dielectric constant of a thin slab substrate. The test equipment consists of a rectangular microstrip antenna, the patch of which is fed either by a center-fed line or an off-center fed microstrip line. By making use of the measured input impedance characteristics or the resonance parameters of a microstrip antenna, the dielectric constant of the substrate to be measured can be easily obtained.

In this paper, a nondestructive method will be reported for measuring the dielectric constant of thin slab substrates. The method is based on the exact solution of the integral equation, and the accurate measurement of the input impedance and the resonant frequency of a rectangular microstrip antenna. Using an analytical formula given in [6], it can be shown that the following expression for the resonant frequency holds:

$$f = c/2a\sqrt{\epsilon_r} \tag{1}$$

where $c = 1/\sqrt{\mu_0 \epsilon_0}$, the velocity of light in free space, ϵ_r is the substrate relative permittivity, and a is the rectangular patch length.

The author has found that the difference between the theoretical and experimental values of the resonant frequency so obtained [6] is about $1 \sim 4$ percent ($\beta h \ll 1$). Furthermore, a few attempts have been made [7], [8] to provide a modified term to the early analytical formula for the input impedance. Based on this result and the measured resonant frequency of a rectangular microstrip antenna, the dielectric constant can be easily obtained.

The actual dielectric constant can be obtained by using the method presented in this paper. Finally, the precision standard cavity resonator method and our method were compared in practical measurements using samples of low-loss materials.

II. Theoretical Determination of the Dielectric Constant

The geometry of the analytical model is illustrated in Fig. 1. The patch is fed either by a center-fed line or an off-center feedline. The origin of the coordinate system is located on the ground plane where the microstrip feedline connects to the patch. The excited voltage on the patch is located at a position where the microstrip feedline connects to the patch. The analytical model gives results which are valid for practical microstrip antennas. To find the integral equation with the patch current as an unknown variable, the antenna system is separated into two regions. Region I is free space having the dielectric constant ϵ_0, and the dielectric constant of the substrate region II is ϵ_r. The problem can be analyzed by first deriving the Hertz

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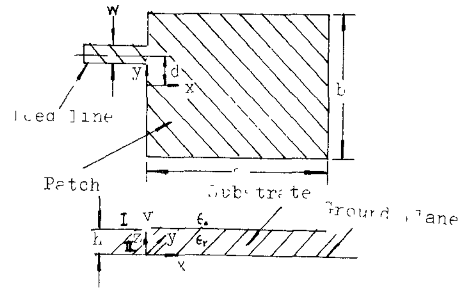

Fig. 1. The structure of a rectangular microstrip antenna.

vectors for the z-axial direction located at regions I and II, and then the Maxwell equations are solved and satisfied with the boundary conditions of the transverse fields located at the plane $z=h$. It should be noted that the analytical model of the rectangular patch antenna for a microstrip feedline has been derived from an integral equation method. In the case of a coaxial feedline, the analytical model for the resonant frequency mentioned above can also be used with the same dimensions of a patch. The following paragraphs describe the various results of the resonant frequency that can be used for two types of feedline.

If we consider only the resonant problem of the dominant mode for the rectangular patch antenna, the integral equation due to a patch current density J can then be found by the Hertz vector as given below [7]:

$$A\bar{J}=\frac{1}{j4\pi^2\omega\epsilon}\iint_{-\infty}^{\infty}\frac{e^{-j\bar{q}\cdot\bar{\Omega}_{xy}}}{\sqrt{\beta^2-|q|^2}}\sin\left(\sqrt{\beta^2-|q|^2}\,h\right)\left\{\frac{\beta}{\cos\left(\sqrt{\beta^2-|q|^2}\,h\right)+j\sqrt{\dfrac{\beta_0^2-|q|^2}{\beta^2-|q|^2}}\sin\left(\sqrt{\beta^2-|q|^2}\,h\right)}\frac{\bar{z}\times\bar{q}\;\bar{z}\times\bar{q}}{|q|^2}\right.$$
$$\left.+\frac{\beta^2-|q|^2}{\cos\left(\sqrt{\beta^2-|q|^2}\,h\right)+\dfrac{j}{\epsilon_r}\sqrt{\dfrac{\beta^2-|q|^2}{\beta_0^2-|q|^2}}\sin\left(\sqrt{\beta^2-|q|^2}\,h\right)}\frac{\bar{q}\bar{q}}{|q|^2}\right\}\cdot\tilde{\bar{J}}\,dq_x\,dq_y$$
$$=\frac{1}{j4\pi^2\omega\epsilon}\iint_{-\infty}^{\infty}\left\{\left(\beta^2\bar{\bar{I}}-\bar{q}\bar{q}\right)\frac{e^{-j\sqrt{\beta^2-|q|^2}\,h}}{\sqrt{\beta^2-|q|^2}}\sin\left(\sqrt{\beta^2-|q|^2}\,h\right)+\frac{\beta^2-|q|^2}{|q|^2}e^{-j\sqrt{\beta^2-|q|^2}\,h}\sin\left(\sqrt{\beta^2-|q|^2}\,h\right)\Big/\sqrt{\beta^2-|q|^2}\right.$$
$$\left.\cdot\left[\sum_{n=1}^{\infty}\left(\frac{\epsilon_r-1}{\epsilon_r}e^{-j\sqrt{\beta^2-|q|^2}\,h}\,j\sin\left(\sqrt{\beta^2-|q|^2}\,h\right)\right)^n\right]\bar{q}\bar{q}\right\}e^{-j\bar{q}\cdot\bar{\Omega}_{xy}}\cdot\tilde{\bar{J}}\,dq_x\,dq_y$$
$$-\frac{h^2}{4\pi^2\omega\epsilon}\iint_{|q|<\beta_0}\left(\frac{\beta^2-\sqrt{\beta_0^2-|q|^2}}{|q|^2}\,\bar{z}\times\bar{q}\;\bar{z}\times\bar{q}+\frac{1}{\epsilon_r}\frac{\left(\beta^2-|q|^2\right)^2}{\sqrt{\beta_0^2-|q|^2}}\frac{\bar{q}\bar{q}}{|q|^2}\right)e^{-j\bar{q}\cdot\bar{\Omega}_{xy}}\cdot\tilde{\bar{J}}\,dq_x\,dq_y$$
$$=-v\delta(x) \qquad (2)$$

where

$\tilde{\bar{J}}=\mathscr{F}\bar{J}$, $\mathscr{F}$ is the Fourier transform,
$\bar{\Omega}_{xy}=\bar{x}x+\bar{y}y$,
$\bar{q}=\bar{x}q_x+\bar{y}q_y$, $|q|^2=q_x^2+q_y^2$,
$\bar{x}$, $\bar{y}$ are unit vectors,
$\omega=2\pi f$,
$\epsilon_r=\epsilon/\epsilon_0$,
$\beta=\sqrt{(m\pi/a)^2+(n\pi/b)^2}$ is the propagation constant,
$\beta_0=2\pi/\lambda$,
$\bar{\bar{I}}=\bar{x}\bar{x}+\bar{y}\bar{y}+\bar{z}\bar{z}$ is the dyad,
h is the substrate thickness, λ is the wavelength,
A is the operator form of $\bar{J}$.

In [2], the first part of the first term is equal to the dielectric material submerged by the whole patch antenna, and the last part of the first term is the modified term. In the equation mentioned above, the second term represents a radiation part. Assuming the condition $\beta h \ll 1$, the following relations exist:

$$\frac{\epsilon_r-1}{\epsilon_r}\left|e^{-j\sqrt{\beta^2-|q|^2}\,h}\,j\sin\left(\sqrt{\beta^2-|q|^2}\,h\right)\right|\leqslant\frac{\epsilon_r-1}{2\epsilon_r}$$

$$\left(\beta^2-|q|^2\right)/|q|^2\doteq-1.$$

From the relations mentioned above, (2) becomes

$$A\bar{J} \doteq \frac{1}{4j\pi^2\omega\epsilon}\left\{\iint_{-\infty}^{\infty}\left(\left(\beta^2\bar{\bar{I}}-\bar{q}\bar{q}\right)\frac{e^{-j\sqrt{\beta^2-|q|^2}h}}{\sqrt{\beta^2-|q|^2}}\cdot\sin\left(\sqrt{\beta^2-|q|^2}h\right)e^{-j\bar{q}\cdot\bar{\Omega}_{xy}}\cdot\tilde{\bar{J}}\,dq_x\,dq_y\right\}$$
$$-\frac{1}{j4\pi^2\omega\epsilon}\iint_{-\infty}^{\infty}\frac{\epsilon_r-1}{\epsilon_r}\bar{q}\bar{q}e^{-j\sqrt{\beta^2-|q|^2}2h}\cdot\frac{\left(j\sin\left(\sqrt{\beta^2-|q|^2}h\right)\right)^2}{j\sqrt{\beta^2-|q|^2}}e^{-j\bar{q}\cdot\bar{\Omega}_{xy}}\cdot\tilde{\bar{J}}\,dq_x\,dq_y$$
$$-\frac{h^2}{4\pi^2\omega\epsilon}\iint_{|q|<\beta_0}\left(\frac{\beta^2\sqrt{\beta_0^2-|q|^2}}{|q|^2}\overline{\bar{z}\times\bar{q}}\;\overline{\bar{z}\times\bar{q}}+\frac{1}{\epsilon_r}\frac{\left(\beta^2-|q|^2\right)^2}{\sqrt{\beta_0^2-|q|^2}}\frac{\bar{q}\bar{q}}{|q|^2}\right)e^{-j\bar{q}\cdot\bar{\Omega}_{xy}}\cdot\tilde{\bar{J}}\,dq_x\,dq_y$$
$$=-v\delta(x). \tag{3}$$

When the width of the feedline is very small (i.e., $\beta w \ll 1$), the current density J on the feedline is constant ($x=0$). In order to obtain the resonant frequency when $n=1$, J is given by

$$J \doteq A_0\beta\sin\left(\beta(a-x)\right)+\frac{\pi}{2a}C_0\cos\left(\frac{\pi}{2a}x\right) \tag{4}$$

where A_0 and C_0 are unknown constants. From (4), A_0 and C_0 cannot be determined. Hence, the integral equation (2) must be solved. Assuming the coupling between the patch and microstrip feedline can be neglected, substituting (4) into (3), we obtain

$$\left(AJ,\beta\sin\left(\beta(a-x)\right)\right)$$
$$\doteq-\frac{h}{j\omega\epsilon}\frac{\pi}{2a}\beta^2b\cos(\beta a)C_0-\frac{h^2}{4\pi^2\omega\epsilon}A_0\epsilon_rb^2\beta^5C_1+\frac{\beta^4}{j4\pi^2\omega\epsilon}$$
$$\cdot\iint_{-\infty}^{\infty}e^{-j\sqrt{\beta^2-|q|^2}h}\frac{\sin\left(\sqrt{\beta^2-|q|^2}h\right)}{\sqrt{\beta^2-|q|^2}}\cdot\left(\left|\widetilde{\sin\left(\frac{\pi}{a}x\right)}\right|^2-\left|\widetilde{\cos\left(\frac{\pi}{a}x\right)}\right|^2\right)dq_x\,dq_yA_0$$
$$-\frac{\beta^4}{j4\pi^2\omega\epsilon}\frac{\epsilon_r-1}{\epsilon_r}\iint_{-\infty}^{\infty}e^{-j2h\sqrt{\beta^2-|q|^2}}\cdot\frac{\left(j\sin\left(\sqrt{\beta^2-|q|^2}h\right)\right)^2}{j\sqrt{\beta^2-|q|^2}}\left|\widetilde{\cos\left(\frac{\pi}{a}x\right)}\right|^2dq_x\,dq_yA_0=0 \tag{5}$$

where $\widetilde{\sin(\pi/ax)}$ and $\widetilde{\cos(\pi/ax)}$ are the Fourier transform of the functions $\sin(\pi/ax)$ and $\cos(\pi/ax)$

$$C_1=\int_0^{2\pi}\int_0^{\pi/2}\left(\epsilon_r\sin^2\theta\cos^2\varphi+\frac{1}{\epsilon_r}\left(\epsilon_r-\cos^2\theta\right)^2\sin^2\varphi\right)\cdot\left(\frac{\sin\left(\frac{\beta_0b}{2}\cos\theta\cos\varphi\right)}{\frac{\beta_0b}{2}\cos\theta\cos\varphi}\right)^2\cdot\frac{2\epsilon_r\left(1+\cos\left(\beta_0a\cos\theta\sin\varphi\right)\right)}{\left(\epsilon_r-\cos^2\theta\sin^2\varphi\right)^2}\cos\theta\,d\theta\,d\varphi. \tag{6}$$

An expression has been derived with the simplification that only the dominant mode propagates, and the functions $\sin(\beta(a-x))$ and $\cos(\beta(a-x))$ in (5) can be replaced by $\sin(\pi/ax)$ and $-\cos(\pi/ax)$, respectively.

In (5), the first term of the integration can be written as

$$\iint_{-\infty}^{\infty}\frac{e^{-j\sqrt{\beta^2-|q|^2}h}}{\sqrt{\beta^2-|q|^2}}\sin\left(\sqrt{\beta^2-|q|^2}h\right)\cdot\left(\left|\widetilde{\sin\left(\frac{\pi}{a}x\right)}\right|^2-\left|\widetilde{\cos\left(\frac{\pi}{a}x\right)}\right|^2\right)dq_x\,dq_y$$
$$=8ab\beta h^2\left(\ln\left(\frac{1}{\beta hr}\right)+2-0.75\frac{b}{a}-\frac{S(\beta b)}{\beta b}\right)\qquad(\beta h\ll 1). \tag{7}$$

In (5), the second term of the integration is given by

$$\iint_{-\infty}^{\infty}e^{-j2h\sqrt{\beta^2-|q|^2}}\frac{\left(j\sin\sqrt{\beta^2-|q|^2}h\right)^2}{j\sqrt{\beta^2-|q|^2}}\left|\widetilde{\cos\left(\frac{\pi}{a}x\right)}\right|^2dq_x\,dq_y$$
$$\doteq-2\pi\int_0^b(b-\eta)\int_0^a\left((a-y)\cos\left(\frac{\pi}{a}y\right)-\frac{a}{\pi}\sin\left(\frac{\pi}{a}x\right)\right)\left(\frac{e^{-j\beta R_1}}{R_1}-\frac{e^{-j\beta R_0}}{R_0}-\frac{e^{-j\beta R_2}}{R_2}\right)dy\,d\eta \tag{8}$$

where

$$R_0=\sqrt{\eta^2+y^2}$$
$$R_1=\sqrt{\eta^2+y^2+4h^2}$$
$$R_2=\sqrt{\eta^2+y^2+16h^2}.$$

Then (8) can be expressed as

$$-2\pi\int_0^b(b-\eta)\left(\int_0^a(a-y)\left(\frac{e^{-j\beta R_1}}{R_1}-\frac{1}{2}\frac{e^{-j\beta R_2}}{R_2}-\frac{e^{-j\beta R_0}}{2R_0}\right)\cos\left(\frac{\pi}{a}y\right)dy\right)d\eta$$
$$\doteq 4\pi bh\left(2\ln\left(\frac{1}{\beta hr}\right)+\frac{a}{b}\ln\left(\frac{b}{2h}\right)-2\left(\frac{a}{b}+1\right)\ln 2+2-0.75\frac{b}{a}-\frac{S(\beta b)}{\beta b}+g(a,b,\beta)\right) \quad (9)$$

where

$$g(a,b,\beta) \doteq 2\left(\frac{4a}{b}-\frac{b}{a}+0.5\frac{a}{\sqrt{a^2+b^2}}-3\sqrt{1+\left(\frac{a}{b}\right)^2}+3\ln\left(\frac{b}{a}+\sqrt{\left(\frac{b}{a}\right)^2+1}\right)-0.5\left(\frac{b}{a}\right)^2\right.$$
$$-\frac{1}{\pi}\left(\frac{2b+a}{2b}-\frac{\pi}{2}-0.5\cos(\beta b)+\frac{\sqrt{a^2+b^2}}{2b}\cos\left(\sqrt{a^2+b^2}\,\beta\right)\right)$$
$$+\frac{a}{2b}\left(1+\frac{2a}{\sqrt{a^2+b^2}}\cos\left(\sqrt{a^2+b^2}\,\beta\right)\right)$$
$$-\pi^2\left(4+0.5\left(\frac{b}{a}\right)^2+2\ln\left(\frac{\sqrt{a^2+b^2}+a}{b}\right)-2\sqrt{1+\left(\frac{b}{a}\right)^2}\quad\ln\left(\frac{2a}{b}\right)-tg^{-1}\left(\frac{b}{a}\right)-\frac{4}{1+\frac{b}{a}+\sqrt{1+\left(\frac{b}{a}\right)^2}}\right)$$
$$-\frac{1}{2\sqrt{\pi}}\int_{\frac{1}{2}\beta b}^{\infty}\frac{\sin t}{\sqrt{t}}dt+\frac{\pi^4}{6}\left(3.5\cdot\ln\left(\frac{b}{a}+\sqrt{1+\left(\frac{b}{a}\right)^2}\right)\right.$$
$$\left.\left.+0.5\frac{b}{a}\sqrt{1+\left(\frac{b}{a}\right)^2}-3\frac{b}{a}-tg^{-1}\left(\frac{b}{a}\right)\right)\right). \quad (10)$$

Substituting (7) and (9) into (5), we obtain

$$-\frac{h}{j\omega\epsilon}\frac{\pi}{2a}\beta^2 b\cdot\cos(\beta a)C_0-\frac{h^2}{4\pi^2\omega\epsilon}A_0\epsilon_r b^2\beta^5 C_1$$
$$+\frac{\beta^2}{j\omega\epsilon}\frac{b}{a}\beta h^2 A_0\left\{\frac{2}{\epsilon_r}\ln\left(\frac{1}{\beta hr}\right)+4-1.5\frac{b}{a}-2\frac{S(\beta b)}{\beta b}-\frac{\epsilon_r-1}{\epsilon_r}\frac{a}{b}\ln\left(\frac{b}{2h}\right)\right.$$
$$\left.+\frac{\epsilon_r-1}{\epsilon_r}\left(\left(4+\frac{2a}{b}\right)\ln 2-2+0.75\frac{b}{a}+\frac{S(\beta b)}{\beta b}-g(a,b,\beta)\right)\right\}=0. \quad (11)$$

From (4) and the boundary condition of the current continuation

$$A_0\beta\sin(\beta a)+\frac{\pi}{2a}\quad C_0=\frac{WJ_0}{b} \quad (12)$$

where J_0 is the current density in a microstrip feedline

$$C_0=\frac{2}{\pi}\sec(\beta a)\cdot\beta h\left\{\frac{2}{\epsilon_r}\ln\left(\frac{1}{\beta hr}\right)-\frac{\epsilon_r-1}{\epsilon_r}\frac{a}{b}\ln\left(\frac{b}{2h}\right)+4-1.5\frac{b}{a}-2\frac{S(\beta b)}{\beta b}\right.$$
$$\left.+\frac{\epsilon_r-1}{\epsilon_r}\left(\left(4+2\frac{a}{b}\right)\ln 2-2+0.75\frac{b}{a}+\frac{S(\beta b)}{\beta b}-g(a,b,\beta)\right\}-j\beta h\frac{2a}{\pi b}C_1\sec(\beta a) \quad (13)$$

and from (11) and (12), the coefficient A_0 is obtained as

$$A_0=(a\cos(\beta a))wJ_0\left[\beta a\cos(\beta a)\cdot\sin(\beta a)+\beta h\left\{\frac{2}{\epsilon_r}\ln\left(\frac{1}{\beta hr}\right)-\frac{\epsilon_r-1}{\epsilon_r}\frac{a}{b}\ln\left(\frac{b}{2h}\right)+4\right.\right.$$
$$-1.5\frac{b}{a}-2\frac{S(\beta b)}{\beta b}+\frac{\epsilon_r-1}{\epsilon_r}\left(\left(4+\frac{2a}{b}\right)\ln 2\right.$$
$$\left.\left.\left.+\frac{S(\beta b)}{\beta b}-g(a,b,\beta)\right)\right\}-j\beta_0 h\frac{a}{b}\left(\frac{b}{\lambda}\right)^2 C_1\right]^{-1}. \quad (14)$$

The analytical formula of the input impedance suitable for accurate determination of resonant frequency of a rectangular patch antenna can be found as follows:

$$z_{in} = -j\eta_0\beta_0 h \frac{\cos\beta a}{\beta b \sin\beta a + \frac{b}{a}\sec(\beta a)\cdot\beta h\cdot f(a,b,\beta,h) - j\beta_0 h(b/\lambda)^2 \sec(\beta a) C_1}$$
$$- j2\eta_0\beta_0 h \sin^2\left(\frac{\pi d}{b}\right)\left(\frac{\sin\frac{\pi w}{2b}}{\frac{\pi w}{2b}}\right)^2$$
$$- j2\eta_0\beta_0 h \frac{\sin^2\left(\frac{\pi d}{b}\right)\left(\sin\left(\frac{\pi w}{2b}\right)/\frac{\pi w}{2b}\right)^2 \cos\left(\sqrt{\beta^2-(\pi/b)^2}\,a\right)}{\sqrt{\beta^2-(\pi/b)^2}\,b\sin\left(\sqrt{\beta_0^2-(\pi/b)^2}\,a\right) - j\beta_0 h(2a/\lambda)^2 \sec\sqrt{\beta^2-(\pi/b)^2}\,C_2}$$
$$- j2\eta_0\beta_0 h\left[\sum_{m=1}^{\infty}\frac{1}{\sqrt{\beta^2-\left(\frac{2m+1}{b}\pi\right)^2}\,b}\cot\left(\sqrt{\beta^2-\left(\frac{2m+1}{b}\pi\right)^2}\,a\right)\left(\frac{\sin\left(\frac{2m+1}{2b}\pi w\right)}{\frac{2m+1}{2b}\pi w}\right)^2\cdot\sin^2\left(\frac{2m+1}{b}\pi d\right)\right.$$
$$\left.+\sum_{n=1}^{\infty}\frac{1}{\sqrt{\beta^2-(2n\pi/b)^2}\,b}\cot\left(\sqrt{\beta^2-(2n\pi/b)^2}\,a\right)\left(\frac{\sin(n\pi w/b)}{n\pi w/b}\right)^2\cos^2(2n\pi d/b)\right] \tag{15}$$

where

$$C_2 = \int_0^{2\pi}\int_0^{\pi/2}\left(\sin^2\theta\sin^2\varphi + \frac{1}{\epsilon_r^2}(\epsilon_r-\cos^2\theta)^2\cos^2\varphi\right)\cdot\left(\frac{\sin\left(\frac{\beta_0 a}{2}\cos\theta\sin\varphi\right)}{\frac{\beta_0 a}{2}\cos\theta\sin\varphi}\right)^2\cdot\frac{1+\cos(\beta_0 b\cos\theta\cos\varphi)}{\left(1-\left(\frac{b}{\pi}\beta_0\cos\theta\cos\varphi\right)^2\right)^2}\cos\theta\,d\theta\,d\varphi \tag{16}$$

$$f(a,b,\epsilon_r,h) = \frac{2}{\epsilon_r}\ln\left(\frac{1}{\beta hr}\right) - \frac{\epsilon_r-1}{\epsilon_r}\frac{a}{b}\ln\left(\frac{b}{2h}\right) + 4 - 1.5\frac{b}{a} - 2\frac{S(\beta b)}{\beta b} + \frac{\epsilon_r-1}{\epsilon_r}\left(\left(4+2\frac{a}{b}\right)\ln 2 - 2 + 0.75\frac{b}{a} + \frac{S(\beta b)}{\beta b} - g(a,b,\beta)\right) \tag{17}$$

where W is the width of a microstrip feedline, and $\eta_0 =$ 377 Ω.

From the expressions mentioned above, the resonant wavelength λ_r is found to be

$$\lambda_r = \Delta\lambda + 2\sqrt{\epsilon_r}\,a. \tag{18}$$

In this formula, the modified value of the resonant frequency $\Delta\lambda$ is expressed as

$$\frac{\Delta\lambda}{\lambda_0} \doteq \frac{1}{\pi^2}\beta h\left\{\frac{2}{\epsilon_r}\ln\left(\frac{1}{\beta hr}\right) - \frac{\epsilon_r-1}{\epsilon_r}\frac{a}{b}\ln\left(\frac{b}{2h}\right) + 4 - 1.5\frac{b}{a} - 2\frac{S(\beta b)}{\beta b} + \frac{\epsilon_r-1}{\epsilon_r}\left(\left(4+2\frac{a}{b}\right)\cdot\ln 2 - 2 + 0.75\frac{b}{a} + \frac{S(\beta b)}{\beta b} - g(a,b,\beta)\right)\right\} \tag{19}$$

where

$$\lambda_0 \doteq 2\sqrt{\epsilon_r}\,a. \tag{20}$$

Based on the measured input impedance characteristics or the resonant frequency of a patch antenna, the dielectric constant can be given as

$$\epsilon_r = ((\lambda_r - \Delta\lambda)/2a)^2. \tag{21}$$

From (19), it follows that the fringing fields would slightly increase the resonant wavelength. In the present method with the fringing effect, it can be seen that the increment of the resonant wavelength is $\Delta\lambda$.

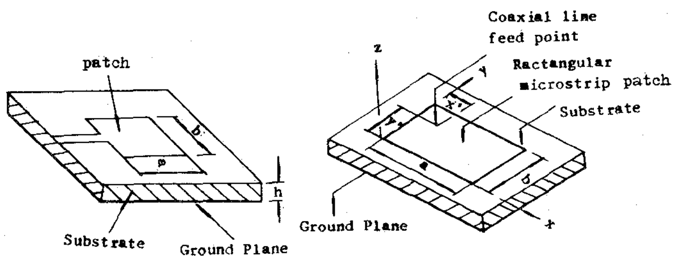

Fig. 2. (a) Rectangular microstrip antenna with microstrip feedline. (b) Rectangular microstrip antenna with coaxial feedline.

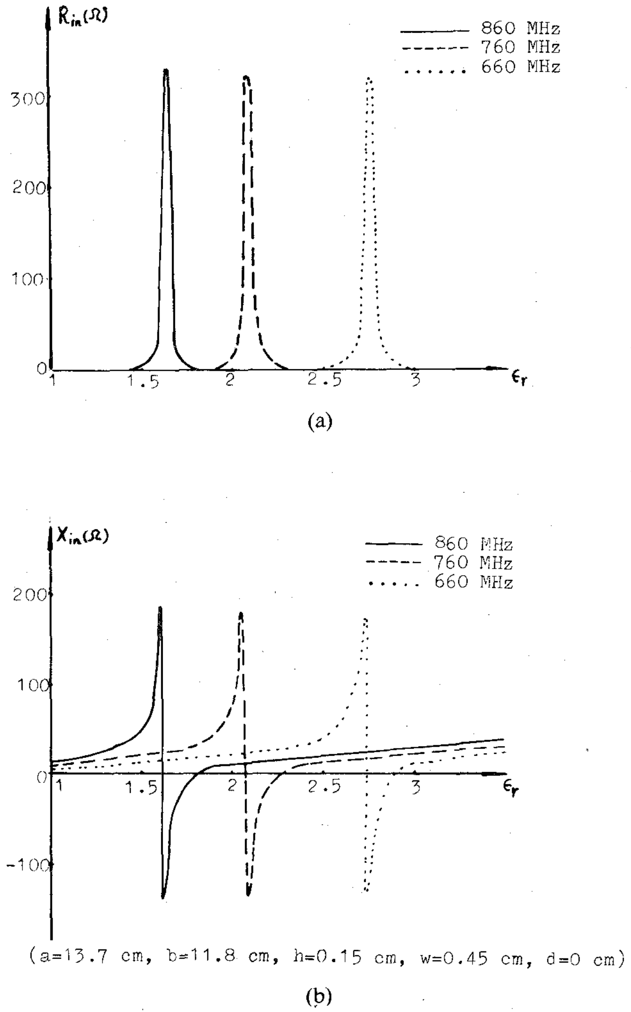

Fig. 3. (a) R_{in} versus ϵ_r. (b) X_{in} versus ϵ_r.

III. Experimental Measurement Procedure

Let us now turn to the measurement procedure to determine the dielectric constant of a thin slab-type material. To this end, a rectangular patch antenna has been constructed and tested. As shown in Fig. 2, the patch antenna we have employed is essentially a radiation cavity. The dielectric constant of the substrate is unknown.

From (15), the input impedance is a function of the permittivity of a substrate, the dimensions of a patch antenna, and the operating frequency

$$Z_{\text{in}} = z(a, b, h, w, d, f, \epsilon_r). \tag{22}$$

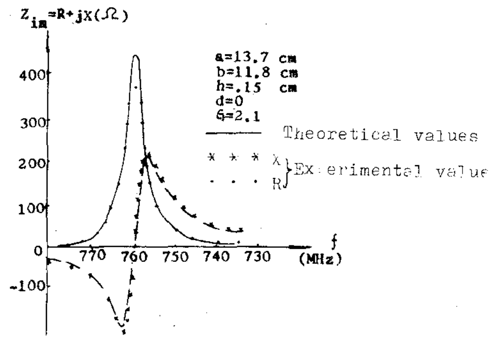

Fig. 4. Input impedance of a rectangular microstrip antenna (center-fed).

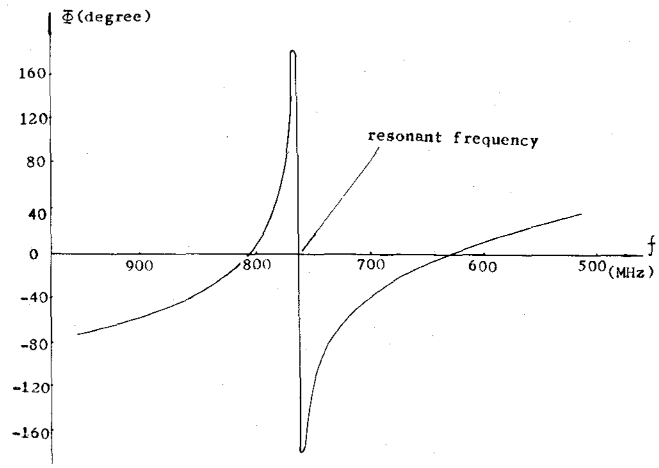

Fig. 5. Phase characteristic of a patch antenna.

The calculated and measured input impedances (i.e., $Z_{\text{in}}t(\epsilon_r)$ and $Z_{\text{in}}e(\epsilon_r)$) as a function of the dielectric constant ϵ_r are shown in Fig. 3. The input impedance is readily measured in the dominant-mode range using an automatic network analyzer. The experimental results are also shown in a display of the real and imaginary parts of the input impedance as a function of frequency. The measured input impedance $Z_{\text{in}}e(f)$ of a patch having dimensions $h = 0.15$ cm, $a = 13.7$ cm, $b = 11.8$ cm, and $d = 0$ cm, are shown in Fig. 4.

The phase characteristic of a patch antenna is shown for the case of $h = 0.15$ cm, $b = 11.8$ cm, $a = 13.7$ cm, and $d = 0$ cm in Fig. 5.

The procedures to determine the dielectric constant of the unknown material may be summarized as follows.

a) Construct test equipment consisting of a rectangular patch antenna. Measure the input impedance $Z_{\text{in}}e(f)$ in the frequency range of the dominant mode.

b) Find the resonant frequency f_r($x_{\text{in}} = 0$, R_{in} is maximum value, $\Phi = 0$ return point) and the resonant wavelength λ_0.

c) Estimate the approximate value of ϵ_r, using (20).

d) Obtain the values of $\Delta\lambda$ and λ_r from (18) and (19).

e) Substitute the obtained values of λ_r and $\Delta\lambda$ into (21) and solve (21) for ϵ_r.

As noted before, the dielectric constant ϵ_r can be measured in the dominant-mode range using the resonant frequency of the patch antenna. At the same time, the dielectric constant ϵ_r can be measured in the dominant-mode range using the following method. The measurements to be made depend on the dielectric constant ϵ_r and the input impedance Z_{in}, i.e., the method described here depends upon (15). At first, we compute $Z_{in}t(\epsilon_r)$ from the known values a, b, h, w, d, f, and plot a set of curves of $Z_{in}t(\epsilon_r)$. Typical plots of $Z_{in}t(\epsilon_r)$ are given in Fig. 3. If the measured frequency f_e and the dimensions of the patch antenna are the same as the resonant frequency and dimensions of a patch antenna on the earlier plotted curves, we note from Fig. 3 that the dielectric constant ϵ_r can be readily obtained.

There is a restriction on these earlier plotted curves. If the measured frequency f_e and the dimensions of the patch antenna are not same, the method is inapplicable. For this reason, we must use the characteristics of the input impedance of the patch antenna.

Therefore, the procedures to determine the dielectric constant of the unknown materials should be also summarized as follows.

a) Choose the patch dimension $a \doteq \lambda_0/2.2$.

b) Choose the patch dimension $b \doteq (1 \sim 1.5)a$.

c) Let $d \geqslant 0$, and $h \ll 1/\beta$.

d) Measure the characteristic of the input impedance as shown in Fig. 4.

e) Find the resonant frequency F_e in the dominant-mode range using an automatic network analyzer.

f) Compute the input impedance $Z_{in}t(\epsilon_r)$ as a function of the dielectric constant ϵ_r as shown in Fig. 3(a) and (b).

g) Obtain the value of ϵ_r from the measured values $Z_{in}e(f)$ (include $R_{in}e(f)$ and $X_{in}e(f)$) and the curves as shown in Fig. 3.

h) Compute the input impedance $Z_{in}t(f)$ as a function of the frequency as shown in Fig. 4, only when given ϵ_r.

i) Compare the measured values of $Z_{in}e(f)$ with the computed values of $Z_{in}t(f)$. If they are not coincident, we change ϵ_r, recompute $Z_{in}t(f)$, and force $Z_{in}e(f)$ to approach $Z_{int}(f)$. Finally, when $Z_{in}e(f)$ and $Z_{in}t(f)$ are coincident with each other, we obtain the accurate value of ϵ_r.

IV. Comparison of Results

Theoretical resonant frequency F_t and ϵ_r for a rectangular patch antenna having parameters h, a, and b are shown in Table I, where the corresponding experimental resonant frequencies F_e are also shown for comparison. Table I also gives some experimental data taken from the indicated references. It should be noted that the theoretical prediction of the resonant frequency using the cavity model [9] requires a semi-empirical correction to give agreement between the theoretical and measured results. For additional verification, the resonant frequency calculated by the present method was compared with that reported in [9]–[13] for two types of feedline.

It can be seen that the present method of analysis is shown to be useful in predicting the resonant frequencies within the tolerance limits of $\Delta F_t \doteq \pm 1.6\%$ ($\beta h \ll 1$).

TABLE I
CALCULATED AND MEASURED VALUES OF THE RESONANT FREQUENCY FOR RECTANGULAR PATCH ANTENNAS WITH DIFFERENT PARAMETERS

parameter / number	h (cm)	a (cm)	b (cm)	ϵ_r	Fe (MHz)	Ft (MHz)	note
1	.15	13.7	11.8	2.08	760	753.3	
2	.2	18.0	12.0	2.08	580	575.3	
3	.15	11.412	7.68	2.08	909	905	
4	.15	7.6	11.4	2.08	1345	1340.1	
5	.159	7.6	11.4	2.62	1202	1195.8	[9]
6	.159	11.4	7.6	2.62	803	810.3	[9]
7	.159	5.95	4.05	2.62	1552	1546.3	[9]
8	.159	5.95	6.05	2.62	1535	1536.5	[9]
9	.159	5.95	8.10	2.62	1524	1524.95	[9]
10	.159	5.95	10.10	2.62	1518	1513.4	[9]
11	.159	5.95	16.05	2.62	1509	1484.1	[9]
12	.159	4.02	4.02	2.55	2275	2291.8	[10]
13	.1524	4.12	4.1	2.5	2228	2249.6	[11]
14	.1524	6.909	11.05	2.5	1344	1342.1	[11]
15	.159	7.62	11.43	2.62	1190	1192.7	[12]
16	.3175	15	7.5	2.56	634	624.1	[12]
17	.159	2.01	2.01	2.55	4425	4510.5	[10]
18	.0794	2.54	5.08	2.59	3570	3523.95	[13]
19	.15875	2.9718	2.9718	8.5	1710	1716.8	[13]
20	.159	11.3	11.2	2.62	815	814.14	[12]

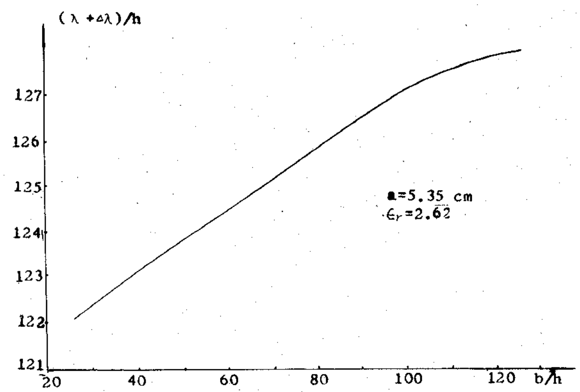

Fig. 6. $(\lambda_0 + \Delta\lambda)/h$ versus b/h.

Figs. 6–8 show the dependence of the $(\Delta\lambda + \lambda_0)/h$ or $\Delta\lambda/h$ on all of the geometrical parameters of the rectangular patch antenna. The comparison of calculated resonant frequencies obtained by the present method with experimental data is shown in Fig. 9. Using the method described above, measured values of the dielectric constant based on (18)–(21) are compared with the corresponding measured values of a waveguide cavity. It can be seen that the measured values of the present method are in agreement with the precision standard cavity resonator method. Table II shows the measured values of the relative dielectric constant by the microstrip antenna method.

Measurements have been carried out on two materials, such as polytetrafluoroethylene and polytetrafluoroethylene with reinforced plastics (Q/HG-13-395-79). Table II

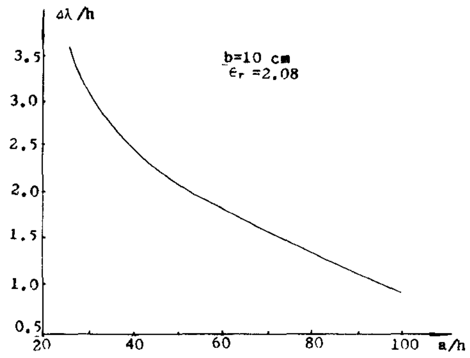

Fig. 7. $\Delta\lambda/h$ versus a/h.

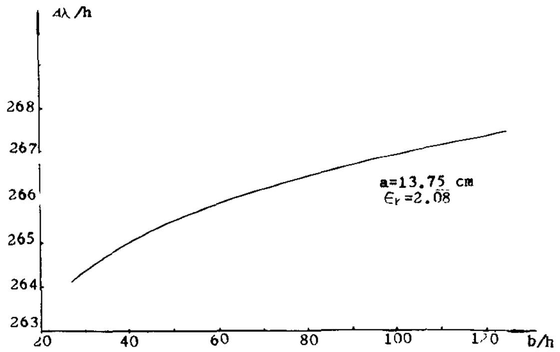

Fig. 8. $\Delta\lambda/h$ versus b/h.

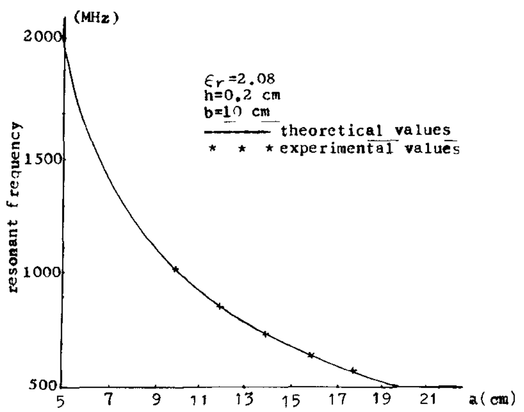

Fig. 9. Resonant frequency of the patch antenna versus patch length.

summarizes the measured values of two different materials. The consistency of the procedure has been verified by employing several different dimensions, and the reliability of the method has been confirmed experimentally. All measurements of the dielectric constant were carried out at ambient temperature.

Using the precision standard cavity resonator method, the measured values of polytetrafluoroethylene are 2.093 ($h = 0.433$ cm) and 2.087 ($h = 0.437$ cm).

It should be noted that the size of the dielectric plates to be measured must be 3 ~ 4 times larger than the dimensions of the rectangular patch to reduce the fringing effect of the patch antenna.

The thickness of the dielectric plate must be much less than $1/\beta$ to reduce the effect of coupling between the patch antenna and feedline. This shows clearly that the effect of the thickness h of the dielectric plate on the measured dielectric constant is small in the case of $h/\lambda < 0.1$. For example, when the wavelength was less than 10 cm, the unknown dielectric constant must, therefore, be less than four. Hence, the method in this paper can not be readily used for the thick dielectric plates. The measurement of the dielectric constant of thick dielectric plates will be reported elsewhere.

TABLE II
MEASURED VALUES OF A RELATIVE DIELECTRIC CONSTANT BY THE MICROSTRIP ANTENNA METHOD

parameter / number	h (cm)	a (cm)	b (cm)	resonant frequency Fe (MHz)	ϵ_r	note
1	.5	13.7	11.6	760	2.074	*
2	.2	18.0	12	560	2.064	*
3	.4	14.8	11.8	700	2.096	*
4	.15	11.8	13.8	500	2.066	*
5	.2	1.97	2.0	5400	2.071	*
6	.2	1.15	2.0	9000	2.1	*
7	.15	7.6	11.4	1198	2.768	**

*The material is polytetrafluoroethylene.
**The material is polytetrafluoroethylene with reinforced plastics.

V. CONCLUSIONS

An analytical expression using an integral equation method is given for the resonant frequency of a rectangular patch antenna. The integral equation formulation is solved for a rectangular patch. A simplified resonant frequency determination with the present analysis method is shown to be useful in predicting the resonant frequency of a rectangular patch antenna and measured dielectric constant of a thin slab-type material. This efficient nondestructive method for measuring the dielectric constant has been explained, and the accuracy of the dielectric constant so obtained is satisfactory.

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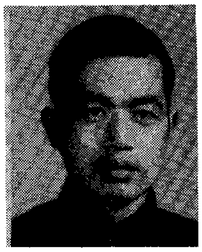

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